# GRAPHICAL METHODS
IN
# REINFORCED CONCRETE DESIGN

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A THESIS presented by

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## INTRODUCTORY

The following pages contain extracts from the manuscript of a treatise on "Graphical Methods of Reinforced Concrete Design". The form and sequence of the articles has been altered to give the discussion as much unity as possible and still cover a number of the applications of the graphical method. The analytical discussions are based upon the formulas given in Turneaure & Maurer's "Principles of Reinforced Concrete Construction" and Prof. A. N. Talbot's Bulletins, issued by the University of Illinois Engineering Experiment Station. These formulas agree with experimental facts, and are now generally accepted.

The diagrams for the graphical solution of formulas contain a number of unique and original features and embody numerous improvements on diagrams previously published. They will enable the designer to solve complex relations occuring in formulas for reinforced concrete beams with comparative ease, and for all values of the terms likely to occur in practice. The principle of the logarithmic chart has been applied to the solving of relations containing more than three variables.

The Plates and discussion as given apply only to

the flexure of beams and slabs, and do not include any mention of shearing or secondary stresses.

The contents, as given by the article headings, are:-

# CHAPTER IV. DEVELOPMENT AND USE OF FORMULAS.

46. RECTANGULAR BEAMS WITH TENSILE REINFORCEMENT.- The underlying principle of reinforced concrete construction is the introduction of steel to take the tensile stresses, leaving the concrete to take the compressive stresses. It follows that in a reinforced beam, the steel is placed on the tension side. The following discussion applies to this simple case, the beam being rectangular in section, and the steel reinforcement being placed in the tension side in the form of rods or bars as shown in Fig.2.

47. Development of Formulas: Rectilinear Variation of Stress in the Concrete.- The notation used in the formulas is given below, and some of these symbols are illustrated in Figs. 1 & 2.

All quantities in inches, inch pounds, or pounds per square inch.

$E_s$ = Modulus of elasticity of the steel.

$E_c$ = Initial modulus of elasticity of the concrete in compression.

n= Ratio of $E_s$ to $E_c$.

$f_s$= Tensile stress in steel.

$f_c$= Compressive stress in extreme fiber of concrete.

$e_s$= Unit elongation of steel.

$e_c$= Unit contraction of extreme fiber of concrete.

b = Breadth of beam or slab.

$d$ = Net depth of beam or slab, i.e., depth from extreme compressive fiber of concrete to the center of gravity of the tensile steel.

$k$ = Ratio of the distance from extreme compressive fiber of concrete to neutral axis- to the net depth "d".

$kd$= Distance from extreme compressive fiber of concrete to neutral axis.

$j$ = Ratio of the distance from the center of compressive stresses in the beam to the center of gravity of the steel- to the total depth "d".

$jd$= Distance from the center of compression in the concrete to the center of gravity of the steel. This quantity is known as the lever arm of the resisting couple or the effective depth of the beam.

$a$ = Cross sectional area of tensile steel in breadth "b".

$A$ = Cross sectional area of one bar or rod.

$s$ = Spacing center to center of bars or rods.

$p$ = Ratio of steel area "a" to area of concrete above steel "bd".

$C$ = Total compressive stress in concrete in breadth "b".

$T$ = Total tensile stress in steel in breadth "b".

$R_s$= "Resistance factor" of the beam or slab based on the steel stress, equals $M_s \div bd^2$.

$R_c$= "Resistance factor" of the beam or slab based on the concrete stress, equals $M_c \div bd^2$.

$M_s$ = Resisting moment of beam or slab as determined by the tensile stress in the steel equals $R_s bd^2$.

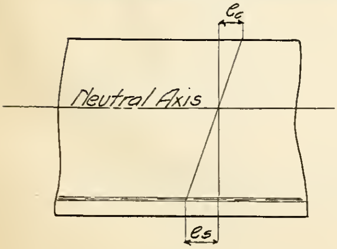
$M_c$ = Resisting moment of beam or slab as determined by the compressive stress in concrete equals $R_c bd^2$.

The formulas which are deduced in the following discussion to express the conditions of stress in a reinforced concrete beam under flexure, are based on certain assumed qualities and methods of behavior of the component materials of the beam. These assumptions are three in number, and are given below. Numbers 1 & 3 are almost universally accepted, but even they are not susceptible to rigid proof. Number 2 is only assumed as an approximation. Formulas based on an assumption more nearly approaching actual conditions will be given later.

(1). Plane sections before flexure remain plane sections during flexure; or, in other words, the deformation in the fibers is proportional to their distance from the neutral axis.

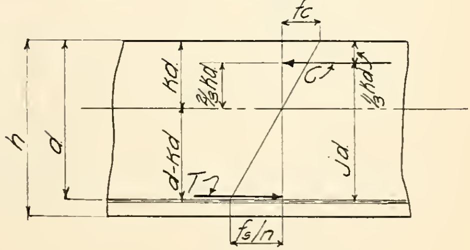
(2). The stress in the concrete varies as the strain.

(3). The concrete has no strength in tension.




STRAIN DIAGRAM

FIG. 1




STRESS DIAGRAM

FIG. 2

The strain diagram, Fig.1, shows the conditions according to assumption (1). This condition is expressed

$$\frac{e_s}{e_c} = \frac{d-kd}{kd} = \frac{1-k}{k} \qquad (1)$$

but $e_s = \frac{f_s}{E_s}$ and $e_c = \frac{f_c}{E_c}$

$$\therefore \quad \frac{e_s}{e_c} = \frac{E_c f_s}{E_s f_c} = \frac{f_s}{n f_c}$$

Substituting this expression in Eq.(1)

$$\frac{f_s}{n f_c} = \frac{1-K}{K} \qquad (2)$$

$$f_s = \frac{n f_c (1-K)}{K} \qquad (3)$$

Eq.(3) gives the stress in the steel, fs, in terms of the stress in the concrete, "fc".

Also from Eq 2

$$K = \frac{n f_c}{n f_c + f_s} \qquad (4)$$

Eq.(4) gives the position of the neutral axis in terms of the stresses "fs" and "fc".

Equating the horizontal stresses acting on the section,

$$T = C$$

$$\text{or } f_s p b d = \tfrac{1}{2} f_c b K d \qquad (5)$$

Substituting for "fs" its value from Eq.(2)

$$\frac{n f_c (1-K) p b d}{K} = \tfrac{1}{2} f_c K b d$$

$$\text{or } 2np(1-K) = K^2$$

Solving the quadratic for "K"

$$K = \sqrt{2pn + p^2 n^2} - pn \qquad (6)$$

Also from Eq.(5)

$$p = \frac{f_c K}{2 f_s} \qquad (7)$$

Substituting for "K" from Eq.(4)

$$p = \frac{n f_c^2}{2 f_s (n f_c + f_s)} \qquad (8)$$

Eq.(6) gives the position of the neutral axis, "K", in

terms of the percentage of steel.

Eq.(8) gives the percent of steel in terms of the stresses in the concrete and steel.

The lever arm of resisting couple, "jd", equals the distance between the center of compression in the concrete and the center of tension in the steel. The center of compression in the concrete is 1/3 "kd" from the top of the beam therefore

$$jd = d - \tfrac{1}{3}kd$$

$$j = 1 - \tfrac{1}{3}k \qquad (9)$$

The resisting moment of the beam may be expressed in terms of the compressive stress in the concrete or of the tensile stress in the steel. It is equal to the "effective depth" "jd" times the total compression or the total tension.

In terms of "fc" it is

$$M_c = jdC$$

$$= \tfrac{1}{2} f_c bd^2 jk \qquad (10)$$

In terms of "fs"

$$M_s = jdT$$

$$= f_s p j bd^2 \qquad (11)$$

Eq.(10) may be written

$$M_c = R_c bd^2 \qquad (12)$$

$$\text{where } R_c = \tfrac{1}{2} f_c k \qquad (13)$$

and Eq. (11) may be written

$$M_s = R_s bd^2 \qquad (14)$$

$$\text{where } R_s = f_s p j \qquad (15)$$

Rc or Rs, the coefficients of $bd^2$, are known as the "resistance factors" of the beam as based on the concrete stress or the steel stress respectively.

48. Use of Formulas.- Plate 1 will enable the designer to solve the foregoing formulas with ease and despatch. Article (49) describes this graphical method. The formulas are simple, however, and with the proper manipulation and a few short cuts may be readily applied to ordinary problems in the design of beams with single reinforcement.

The designer will meet with two general problems; (1).Given the allowable unit stresses in the materials, the span,and the loading to determine the dimensions of the beam and the amount of reinforcement. (2).Given the dimensions of the beam, amount of reinforcement, and the loading, to find the unit stress in the concrete and in the steel. The first case may be called the direct problem and the second case the inverse problem.

(1). The first step in the direct problem is to find the external bending moment which the beam must be designed to resist. The moment is designated by "M". The moment of resistance of the beam must equal the external bending moment, or

$$M_c = R_c bd^2 = M \qquad M_s = R_s bd^2 = M$$

and $bd^2 = \dfrac{M}{R_c}$ also $bd^2 = \dfrac{M}{R_s}$

In order to find the value of "$bd^2$" the expression "$R_s$" or "$R_c$" must be calculated.

When the only given quantities are the stresses "$f_s$" and "$f_c$", the percent of steel may be derived from Eq.(8). For this percent of steel "$R_s$" will equal "$R_c$" and either one may be used in solving for "$bd^2$". Using "$R_s$", which equals "$f_s pj$", "$f_s$" is given, "$p$" follows from Eq. (8) and "$j$" follows from Eq.(4) or (6), together with Eq.(9). "$bd^2$" is then obtained from "$bd^2 = M \div R_s$". Either dimension of the beam, "b" or "d", may be assumed from the requirements of the situation or from the considerations discussed in Chapter--- and the remaining dimension figured. The area of steel is determined from this cross-section and the percent of reinforcement, and the steel is distributed according to the considerations discussed in Art.-- Chapter---.

(2). The calculation of the external bending moment is also the first step in the inverse problem. The percent of reinforcement can be figured from the sectional area of the steel and the beam size as given. $R_c$ is then obtained from the equation "$M = R_c bd^2$".

$$R_c = \tfrac{1}{2} f_c K j$$

$$\therefore \quad f_c = \frac{2R_c}{Kj}$$

K and j are given by Eqs.(6) and (9). The stress in the steel can now be found from Eq.(3).

49. <u>Graphical Solution of Formulas: Construction of Diagrams</u>.- All of the operations described in the pre-

ceding article may be readily performed, and sufficiently accurate results obtained, by use of the diagrams on Plate 1. The construction of these diagrams will be described before their use is explained.

Referring to Plate 1, the lower part shows values of "p" plotted along the horizontal axis, and values of "R" plotted along the vertical axis, both to a logarithmic scale. The curves marked "Values of "fc" are the graphs for various values of "fc" of the equation

$$R_c = \frac{1}{2} f_c (\sqrt{2pn+pn^2} - pn)(1-\frac{1}{3}[\sqrt{2pn+pn^2} - pn])$$

which is derived from Eq.(13)

$$R_c = \frac{1}{2} f_c k j = \frac{1}{2} f_c k (1 - \frac{1}{3}k)$$

by substituting for "K" its value in terms of "p" from Eq.(6). The curves marked values of "fs" are graphs for various values of "fs" of the equation

$$R_s = f_s p (1 - \frac{1}{3}[\sqrt{2pn+pn^2} - pn])$$

which is derived from Eq.(15).

$$R_s = f_s p j = f_s p (1 - \frac{1}{3}k)$$

by substituting for "k" its value in terms of "p" from Eq.(6).

At the top of Plate 1 are two single curves. The one marked "Values of k" is the graph of Eq.(6), the values of "p" are written along the horizontal axis, and the values of k along the vertical axis to the right, both to a logarithmic scale. The other curve marked "Values of j" is the graph of the equation

$$j = 1 - \frac{1}{3}k = 1 - \frac{1}{3}(\sqrt{2pn+pn^2} - pn)$$

the values of "p" being written along the horizontal

axis and the values of "j" along the vertical axis to the left, also to a logarithmic scale.

In plotting the curves on Plate 1, a number of values of "p" were assumed. From Eq.(6) values of "k" were calculated corresponding to these values of "p"; and from Eq.(9) values of "j" were calculated corresponding to these values of "k" and "p", The curves for "k" and "j" were then plotted as shown.

Next, the curve for fs = 10000 was plotted by calculating values for "$R_s$" from the equation $R_s$ = fspj = 10000 pj, corresponding to the values of "p" and "j" used for the "j" curve at the top of the plate. In the same way the curve for fc = 200 was plotted by calculating values for "$R_c$" from the equation $R_c$ = 1/2 fckj = 100 Kj, corresponding to the values of "k" and "J" ( and therefore "p"), used in plotting the "k" and "j" curves at the top of the plate. The "fc" curves, plotted to a logarithmic scale are parallel, as are also the "fs" curves. Therefore, in plotting the curves for other values of fc or fs, it was only necessary to find a single point and draw a line thru the point parallel to the curve already plotted. Because of this parallelism of the curves it becomes a simple matter for the designer to plot new curves for any values of "fc" or "fs" he may desire.

The abscissa of the point of intersection of any

"fs" curve with any "fc" curve gives the percent of steel for which the resisting moments "$R_s$" and "$R_c$", based on the two unit stresses are equal. Or from another point of view, it may be said that this intersection marks the bending moment which will just produce the stresses "fs" and "fc".

The proof of the parallelism of the "fc" and "fs" curves when drawn to a logarithmic scale is as follows. The equation of any "fc" curve is

$$R_c = \tfrac{1}{2} f_c \left(\sqrt{2pn + \overline{pn}^2} - pn\right)\left(1 - \tfrac{1}{3}\left[\sqrt{2pn + \overline{pn}^2} - pn\right]\right)$$

and when drawn to a logarithmic scale this becomes

$$\log R_c = \log \tfrac{1}{2} f_c + \log\left(\sqrt{2pn + \overline{pn}^2} - pn\right) + \log\left(1 - \tfrac{1}{3}\left[\sqrt{2pn + \overline{pn}^2} - pn\right]\right)$$

For different values of "fc" the only change in the above equation is in the term log.1/2fc which is the term which does not contain either of the two variables "$R_c$" or "fc". It is a theorem of analytical geometry that equations which differ by a constant quantity are equations of parallel curves, hence the "fc" curves, which fulfill this condition, are parallel.

The same proof of parallelism holds for the fs curves which are plots of the equation

$$\log R_s = \log f_s + \log p + \log\left(1 - \tfrac{1}{3}\left[\sqrt{2pn + \overline{pn}^2} - pn\right]\right)$$

The "fs" and "fc" curves, when plotted to the ordinary arithmetical scale, will all meet at the origin of coordinates, as may be seen from an inspection of the equations. However, when the logarithmic scale is used the origin of coordinates is removed to infinity,

since the logarithm of zero is infinity; in other words, as the curves meet at infinity, they must be parallel.

50. Graphical Solution of Formulas; Use of Diagrams.- In Article (48) were stated the two general problems which confront the designer and checker of reinforced concrete beams. These problems can be readily solved by means of the diagram on Plate 1.

Taking the direct problem first, when the allowable stresses in the concrete and in the steel are specified, the curves for the given concrete and steel stressed are noted, and their intersection located. The abscissa of this point of intersection is the percent of steel to be used to give the required stresses and the ordinate is the resistance factor of the beam. Dividing the external bending moment by this factor, the product "$bd^2$" is obtained, from which the dimensions of the beam may be established.

The solution of the inverse problem is even more simple. Having given the external bending moment, the dimensions of the beam, and the percent of steel, the first step is to divide the moment by "$bd^2$" and obtain the required resistance factor. With this resistance factor as the ordinate and the given percent of steel as the abscissa, a point on the diagram is located. The "fc" and "fs" curves which intersect at this point give the stresses in the concrete and in the steel. If there are no curves intersecting at this point, the stresses

must be judged by interpolation.

The "k" and "j" curves will not be used very often in graphical design.

<u>Examples</u>:-

(1). To design a culvert cover for a culvert 8 feet wide and 6 feet high, the cover being 16 feet under a highway. The allowable compressive stress in the concrete to be 700 lbs. per sq. in. and the allowable tensile stress in the steel is 14,000 lbs. per sq. in.

The maximum bending moment on the culvert top for a width of 12 inches is found to be 153,600 in. lbs. From the diagram on Plate 1, it is noted that the curves for fc = 700 and fs = 14,000 intersect where R= 115 and "p"= .0095. Dividing the moment 153,600 in. lbs. by 115 gives "$bd^2$" = 1336, Since "b" = 12, "$d^2$" = 111.3, and "d" = 10.5 ins. The area of steel per 12 in. width = .0095x12x10.5=1.2 sq.ins.

(2). To find the unit stresses in the steel and concrete in a beam 12 ins. deep, 10 ins. wide, reinforced by 4-1/2 in. square bars, imbedded 2 inches from the bottom under a bending moment of 60,000 in. lbs.

The steel ratio, "p", equals 4×¼÷10×10=.01. The required resistance factor equals 60,000 ÷ $bd^2$ or 60,000 ÷ 10 X $10^2$ = 60. Referring to the diagram, Plate (1), the point is located vertically above

p = .01 and horizontally on a line with R = 60. By interpolating, the point is found to be about at the intersection of a curve for fc = 360 and fs = 6900. These values of "fc" and "fs" are the required stresses in the concrete and steel respectively.

51. *Talbot's Theory for the Variation of Stress in the Concrete.*- Concrete is similar to cast iron and timber in that it does not obey Hooke's Law of the proportionality of stress to strain. In other words, the curve which expresses the relation between stress and strain is not a straight line. It was assumed in the foregoing discussion that this curve was straight. This is very nearly the case for low stresses, so nearly in fact, that the rectilinear variation may be used in all calculations for working loads. For high concrete stresses approaching the ultimate, however, the stress-strain variation is far from being rectilinear, the plotted results of many experiments carried to the crushing strength of the concrete indicating that it can be expressed by a parabola. The top of this parabola (marked C in Fig. 3) is the vertex when the concrete is stressed to its ultimate value; for stresses below the ultimate the parabola is not complete.

52. *Development of Formulas: Talbot's Theory.*- The notation used in the following discussion is the same as that used in the formulas for rectilinear variation

with the addition of the following symbols:

$f'_c$ = The ultimate compressive stress in the extreme fiber of the concrete.

$e'_c$ = Unit deformation in the extreme fiber of the concrete when at its ultimate compressive stress.

$q$ = The ratio of any strain to the ultimate deformation, $q = \frac{e_c}{e'_c}$ ,

The assumptions used in the following derivations are:

(1). Plane sections before flexure remain plane sections during flexure; or, in other words, the deformation in the fibers is proportional to their distance from the neutral axis.

(2). The ratio of the unit deformation to the unit stress in the concrete is variable, the relation being expressed by a parabolic curve which becomes a full parabola when the ultimate compressive stress is reached.

(3). The concrete has no strength in tension.

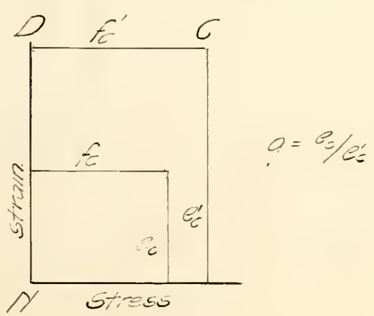



Fig. 3.

In deriving the equations based on the above assumptions, there are three formulas necessary which depend for proof on the properties of a parabola. These formulas are developed in Appendix II.

(a). The relation of the abscissa, "$f_c$", at any point to the abscissa, "$f_c'$" at the vertex is expressed

$$\frac{f_c}{f_c'} = q(2-q) \qquad (A)$$

$$q = 1 - \sqrt{1 - \frac{f_c}{f_c'}} \qquad (A')$$

(b), The average abscissa of the parabolic area ABN (Fig. 3.-) is equal to

$$\frac{3-q}{3(2-q)} f_c \qquad (B)$$

(c). The distance to the center of gravity of the parabolic area ABN from the upper line AB equals

$$\frac{4-q}{4(3-q)} kd$$

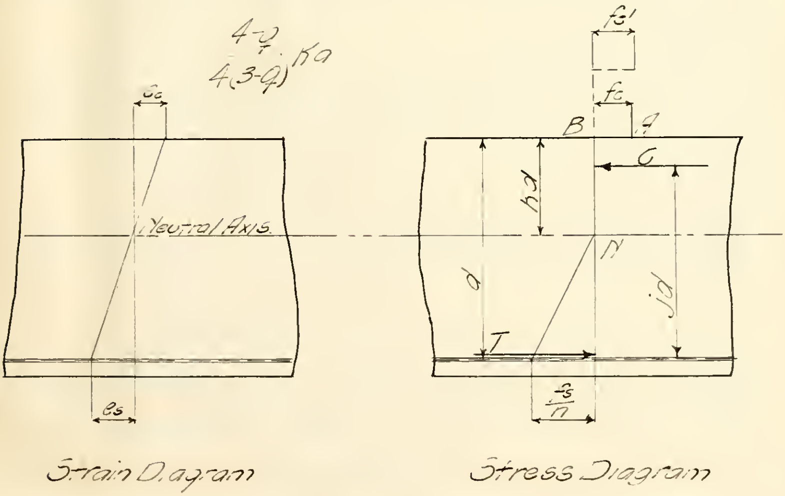



Fig. 4.

From assumption (1) and the strain diagram, Fig. 4.

$$\frac{e_s}{e_c} = \frac{d-kd}{kd} \qquad (1)$$

but $e_s = \frac{f_s}{E_s}$

and $e_c = q e'_c = q \frac{f'_c}{E_c} = \frac{f_c}{E_c(1-\frac{q}{2})}$ from Eq. A

therefore $$\frac{e_s}{e_c} = \frac{f_s E_c(1-q/2)}{E_s f_c} = \frac{d-kd}{k} \qquad (2)$$

and $$f_s = \frac{n f_c(1-k)}{k(1-\frac{1}{2}q)} \qquad (3)$$

also from Eq. 2 $$k = \frac{n f_c}{f_s(1-\frac{q}{2}) + n f_c} \qquad (4)$$

Equating the total tension to the total compression on the section, and referring to Eq. 3 above.

$$f_s p b d = \frac{3-q}{3(2-q)} f_c b k d \qquad (5)$$

Substituting for "$f_s/f_c$" from Eq. (2)

$$6pn(1-k) = k^2(3-q)$$

$$k = \sqrt{2\frac{3pn}{3(2-q)} + \left(\frac{3pn}{3-q}\right)^2} - \frac{3pn}{3-q} \qquad (6)$$

Also from Eq. (5), by substituting for "k" from Eq. (4)

$$p = \frac{3-q}{3(2-q)} \cdot \frac{n f_c^2}{f_s\left(f_s(1-\frac{1}{2}q) + n f_c\right)} \qquad (7)$$

The lever arm of the resisting couple, "j", is equal to the difference between the total depth, "d", and the distance from the extreme compressive fiber to the centroid of the compressive stresses as given in Eq. C

$$jd = d - \frac{4-q}{4(3-q)} kd$$

$$j = 1 - \frac{4-q}{4(3-q)} k \qquad (8)$$

The resisting moment of the beam expressed in terms of the concrete stress is

$$M_c = Cjd = \frac{3-q}{3(2-q)} f_c k j b d^2$$

Expressed in terms of the steel stress, the resisting moment equals

$$M_s = Tjd = f_s A j d = f_s p j b d^2$$

53. Use of Formulas:- The formulas just derived according to Talbot's Theory are more complicated than the formulas for the straight-line theory, due to the presence of "q". For this reason the graphical method of Art. 54 will be generally used. In solving the inverse problem, it will be noticed that the analytical method becomes so difficult, due to the tedious calculations, as to be almost prohibitive.

The direct problem stated in Art. 45 can be solved as follows: From Eq. A' the value of "q" may be determined, $f''_c$ being given and $f'_c$ being assumed, according to the kind of concrete to be used. Then, having calculated $M'$,

$$M' = R_s b d^2 = f_s p j b d^2$$
$$bd^2 = M' / f_s p j$$

$f''_s$ is given, "p" may be determined from Eq.(7), and "j" may be determined from Eqs.(8) and (4). Hence, $bd^2$ and the dimensions of the beam may be figured.

In the inverse problem, it is the stresses which are required and this fact makes the solution difficult

because the value of "q" depends on the concrete stress, and "q" appears in all the formulas which must be used to obtain this stress. The method of procedure is; from the given bending moment, "$R_c$" is obtained from the relation

$$M = R_c b a^2$$

$$\text{But } R_c = \frac{3-q}{3(2-q)} j K f_c$$

$$\text{Hence } f_c = \frac{3(2-q)}{3-q} \cdot \frac{R_c}{jK} \qquad (S)$$

This equation is the one which must be used to solve "$f_c$", the concrete stress, and as may be seen, it contains two interdependent variables "$f_c$" and "q". As is usual in such equations, one variable is guessed at and the other figured; in this case "q" is assumed and $f_c$ is figured. After $f_c$ is obtained, "q" is calculated by Eq. A'; this calculated value is substituted in Eq. (S), and the process is repeated until the assumed and calculated values of "q" agree. Then "$f_s$" is derived from Eq. (3)

54. <u>Graphical Solution of Formulas</u>:- <u>Construction of Diagrams</u>.- The difficulties which have just been mentioned in the analytical solution of formulas based on Talbot's theory, may be readily overcome by adopting a graphical method, using the diagram on Plate II. This diagram is constructed on the same general plan as the one to be found on Plate I, which has been described in Art. 50.

The lower part of Plate II. shows a number of curves for various values of "$f_s$" and "$f_c$". The curves for "$f_s$" are graphs of an equation, in its logarithmic form, in terms of "$f_s$", "p", "$R_s$" and "$f_c$'" derived from the equation

$$R_s = f_s pj$$

by substituting for "j" from Eqs. (8), (6), (A') and (3), Art. 52. This derived equation has three variables; "$R_s$" and "p" plotted on the axes of coordinates and "$f_s$" chosen constant for any particular curve. The curves marked "$f_c$" are graphs of an equation in logarithmic form, in terms of "$f_c$", "p", "$R_c$" and "$f_c$", derived from

$$R_c = \frac{3-q}{3(2-q)} f_c kj$$

by substituting for "j", "k", and "q" from Eqs.(8),(6), and A'. Of the three variables, "$f_c$", "p", and "$R_c$", "$f_c$" is chosen constant for any particular "$f_c$" curve and "p" and "$R_c$" are plotted on the axes of coordinates.

The mathematical proof of the parallelism of the "$f_s$" curves and the "$f_c$" curves, is analagous to that given in Art.49 for the straight-line theory. The intersection of the "$f_s$" and "$f_c$" curves are here not on a logarithmic scale of "R" because of the variable factor "q". For a constant value of "p" in the equation $R = f_s pj$, "R" varies as "$f_s j$". But unlike the straight-line formula "j" is here a function of "q"

and varies inversely as same, there being about 5 per cent difference between the values of "j" for "q" = 1 and "q" = 0. Hence the maximum error possible is 2-1/2 per cent, provided one intermediate "$f_s$" curve is plotted for the correct value of "j" & "q".

In the equation for any "$f_c$" curve "k" and "j" are both functions of "q". As before stated "j" varies inversely as this function, with a maximum variation of 5 per cent. "k" however, varies directly as this function of "q" with a maximum variation of 10 per cent. Hence there is a maximum resultant variation in "R" due to "kj" of 5 per cent, or the same as for the "$f_s$" curves.

Thus if we assume "k" and "j" constant, the maximum error on either side of a correctly plotted intermediate curve will here also be only 2-1/2 per cent, and "R" then may be said to vary, as $\frac{3-q}{3(2-q)}$ . The intersections of the parallel "$f_c$" curves on any vertical (logarithmic scale) may be calculated from the last expression by substituting corresponding values of "q" and "$f_c$". The curves of Plate II. are free from any approximations and may be used with confidence, as they are correctly plotted for all values of "q".

At the top of Plate IV. are curves for values of "j" and "k" for "$f_c$" = 700. These curves are graphs of Eqs. (8) and (6). The value of "q" in these equations corresponds to "$f_c$" = 700, and "$f_c$'" = 2000.

55. <u>Graphical Solution of Formulas:- Use of Diagrams</u>.- The explanation of the use of the diagrams on Plate I., made for the theory of straight-line variation, given in Art. 50, applies to Plate II., the only difference in the two plates being the difference in the position of the curves, due to the difference in the theories.

<u>Determination of Factors of Safety</u>.- Prof. Talbot's theory for the variation of stress in the concrete holds for all stresses up to the ultimate and in this way affords a method for determining the actual factor of safety of any intermediate stress. The strength in flexure of a reinforced concrete beam of given dimensions depends on its "resistance factor", and this factor is a function of the unit stresses in the concrete and the steel. If certain working stresses are assumed, and a working resistance factor determined, this factor is a measure of the safe strength of the beam. The ratio of this safe strength to the ultimate strength, equals the ratio of the working resistance factor to the ultimate resistance factor, and this ratio is the factor of safety of the working stresses.

Plate II. gives values of the resistance factors for any combination of stresses and this affords a

a ready method for finding the factor of safety of such stresses. For example, given working stresses on the steel and concrete of 19,000 #/ sq. in. and 800#/ sq. in. respectively, an elastic limit of steel equal to 55,000#/ sq. in. and an ultimate strength of concrete equal to 2,000#/sq. in.:-to find the factor of safety of the design. On Plate II. opposite the intersection of the steel and concrete curves for 19,000 and 800 respectively, find "p" = .009 and "R" = 145, traversing vertically to the curve for "$f_c$" = 2,000 find "R" = 520 and "$f_s$" = 70,000. This shows that the factor of safety as far as the concrete is concerned is 520 ÷ 145 = 3.6 but that the actual controlling factor depends on the steel because the beam is weakest in tension. Following the vertical for "p" = .009 to the steel curve for 55,000 the value of "R" is found to be 410. Hence the actual factor of safety of the beam designed according to the working stresses specified, is 410 ÷ 145 = 2.8.

56. RECTANGULAR BEAMS WITH COMPRESSIVE & TENSILE REINFORCEMENT.- One of the fundamental advantages of reinforced concrete construction over all-steel construction is due to the fact that, in the former, the cheaper material is utilized to take the compressive stresses, concrete being cheaper when used in compression than steel even when the difference in strength is considered. Because of this fact, reinforced concrete beams are not reinforced for compression when there are no restrictions as to dimensions. When the depth of the beam is limited, however, it is often found necessary to carry part of the compressive stresses by steel reinforcement and thus decrease the total concrete compressive area required. There are numerous cases in engineering construction where shallower beams are necessary than can be obtained by the use of single reinforcement, the most general occuring in bridge and culvert construction where the grade of the roadway or tracks is fixed and a maximum headroom or waterway under the bridge is desirable.

Steel reinforcement, on the compression as well as the tension side also occurs in beams which are subjected to a reversal of bending moment, As an example may be mentioned continuous beams.

In continuous beams of short spans which are reinforced to take both positive and negative bending moment, the tension steel for one kind of moment is

often made continuous between points having the same sign of moment, and thus serves as compressive steel for the points having the opposite moment. Such continuous beams occur most frequently in building construction, and there it is usually the tensile reinforcement from the middle of the span, continued thru the points of negative moment at the columns, which furnishes the compressive reinforcement for the beam at these points.

Double reinforcement is always used in the walls between adjacent reservoirs or basins where the alternate emptying and filling makes it necessary for the walls to act as cantilevers in either direction. Such reinforcement is also used in the sides of bins and tanks built in clusters, where the alternate emptying and filling causes the sides to act as slabs loaded on either side. Columns, piers, walls and arch rings are ordinarily designed with double reinforcement but the bending moments in these cases are combined with direct thrusts which require a different analysis. (See Flexure and Direct Stress. Art.-- )

Following is given the development of formulas for beams with double reinforcement and a discussion of their use in practical design. Plates (III) and (IV) and accompanying explanations give graphical solutions

of the formulas which will save time and labor and give results sufficiently accurate for all purposes.

57. Development of Formulas. The notation adopted for beams with single reinforcement, (see page 24), with the addition of the following symbols, is used in these formulas. Fig. 5 is a diagram showing a number of the symbols graphically.

d'= Distance from the extreme compressive fiber of the concrete to the center of gravity of the compressive steel.

x = Distance from the extreme compressive fiber of the concrete to the resultant of the compressive stresses in the beam,- both in the concrete and the steel.

a'= Cross sectional area of compressive steel in breadth "b".

p'= Ratio of area of compressive steel, "a'" to area of concrete above tensile steel, "bd".

fs'= Unit stress in compressive steel.

T = Total tension in steel in section in breadth "b".

C'= Total compressive stress in steel in breadth "b".

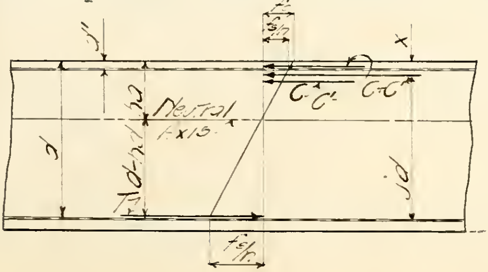


FIG. 5.

The theory of the rectilinear variation of stress in the concrete is adopted as well as the other assumptions given on page (4) for beams with single reinforcement.

The deformations being proportional to the distances from the neutral axis and equal to the corresponding stress divided by the coefficient of elasticity,

$$\frac{e_s}{e_c} = \frac{d-Kd}{Kd} = \frac{1-K}{K} \quad \text{but} \quad \frac{e_s}{e_c} = \frac{E_c f_s}{E_s f_c} = \frac{f_s}{nf_c}$$

therefore

$$\frac{f_s}{nf_c} = \frac{1-K}{K}.$$

and

$$f_s = \frac{nf_c}{K}(1-K) \qquad (1)$$

which gives the tensile stress in the steel in terms of the compressive stress in the concrete.

Similarly

$$\frac{f_s'}{nf_c} = \frac{Kd-d'}{Kd}$$

$$f_s' = nf_c \frac{Kd-d'}{Kd} \qquad (2)$$

also

$$\frac{f_s'}{f_s} = \frac{Kd-d'}{d(1-K)}$$

$$f_s' = f_s \frac{K-d'/d}{1-K} \qquad (3)$$

From Eq.(1) an expression for "k" may be derived

$$K = \frac{nf_c}{nf_c+f_s} \qquad (4)$$

From Eq.(2) another expression for "k" may be derived

$$K = \frac{fs' + nfca''}{nfca} \qquad (5)$$

Equating the horizontal stresses acting on the section of the beam

$$T = C + C'$$

$$f_s p b d = \tfrac{1}{2} f_c b K d + f_s' p' b d \qquad (6)$$

Substituting values for "fs" and "fs'" from Eqs.(1) & (2) and writing "pbd" for "A" and "p'bd" for "A'",

$$K^2 + 2n(p+p')K - 2n(p+p'\tfrac{d''}{d}) = 0$$

$$K = \sqrt{n^2(p+p') - 2n(p+p'\tfrac{d''}{d})} - n(p+p') \qquad (7)$$

This is the formula giving the position of the neutral axis for double-reinforced beams.

Also from Eq.(6), by substituting for "k" from Eq.(4)

$$p = \frac{\tfrac{1}{2} n f_c^2}{n f_c f_s + f_s^2} + \frac{f_s'}{f_s} p' \qquad (8)$$

From Eq.(6) by substituting for "k" from Eq.(5)

$$p = \frac{\tfrac{1}{2}(fs' + nfcd'')}{ndfs} + \frac{fs'}{fs} p' \qquad (9)$$

The position of the resultant of the compressive stresses is found by taking moments about the top of the section.

$$x(C+C') = d'C' + \tfrac{1}{3}KdC$$

$$x = \frac{d'C' + \tfrac{1}{3}KdC}{C+C'}$$

$$x = \frac{\tfrac{d'}{d}\tfrac{C'}{C} + \tfrac{1}{3}K}{1 + \tfrac{C'}{C}}\, d \qquad (10)$$

$$\frac{C'}{C} = \frac{f_s' A'}{\tfrac{1}{2} f_c K b d} = \frac{2p' f_s'}{f_c K} \qquad (11)$$

Substituting for "fs'" from Eq.(2) this becomes

$$\frac{C'}{C} = \frac{2p'n(k - a/d)}{k^2} \qquad (11)$$

Finally, by definition, the effective depth of the beam, "jd" = "d-x" or

$$j = 1 - x/d$$

The resisting moment of the beam may be expressed in terms of the compressive stress or the tensile stress. In terms of the former it is

$$M_c = jd(C + C') = (d - x)(C + C')$$

Substituting the value for x from Eq.(10), the value for "fs'" from Eq.(2), and writing "C" = "1/2 fckbd" and "C'" = "2p'fs'"

$$M_c = \left[k(1/2 - 1/6\,k) + \frac{np'}{k}(k - a/d)(1 - a/d)\right] f_c b d^2 \qquad (13)$$

In terms of the tensile stress in the steel

$$M_s = jdT \qquad (14)$$
$$= f_s p j b d^2$$

58. Use of Formulas.- The designer who has much occasion to solve problems in double reinforcement will undoubtedly use the diagrams on Plate III, following the graphical method outlined in the next article, the foregoing formulas being too cumbersome for ready manipulation. The following suggestions are given, however, in case the designer prefers to check his work analytically.

The two problems, direct and inverse, described in Article 48 in connection with the use of formulas

for single reinforcement present themselves in the same form. In solving the former, the external bending moment is first calculated. Then "p" and "p'" are computed from the simultaneous equations (8) and (9). This latter process is tedious and the designer is advised to use the approximate method for determining the amount of compressive reinforcement described in Article 61. The external moment equals the resisting moment

$$M - M_s = R_s bd^2 = f_s p j bd^2$$
$$bd^2 = \frac{M}{R_s} = \frac{M}{f_s p j}$$

To get a value for "$bd^2$" the value of "fs","p", and "j" must be known. "fs" is given, "p" has been figured, and "j" may be determined from Eqs.(12), (10), and (11). This value of "$bd^2$" gives the dimensions of the beam.

In the inverse problem, "M","p","p'", "b" and "d" are given. "$R_s$" is obtained from the equation "$R_s$" = "M $bd^2$". Since "$R_s$" = "fspj", "fs" = "$R_s$" ÷ "pj". In this equation "R" and "p" are known and "j" may be calculated from Eqs.(12),(10), and (11). Having found the value for "fs" in this fashion, "fc" and "fs'" follow from Eqs.(1) & (2).

59. <u>Graphical Solution of Formulas: Construction of Diagrams.</u>- Diagrams which give a graphical solution for problems in double-reinforcement are to be found on Plate III.

In the construction of the diagrams, "d'/d" was assumed as 1/10 and "n" as 12. All quantities were plotted to a logarithmic form. Quadrant I has values of "k" plotted

along the vertical axis and values of "$f_s$" plotted along the horizontal axis. The curves shown solid are graphs of Eq. (2), Art. 57, for various values of "$f_s$", and the curves shown dotted are graphs of Eq.(3), Art. 57, for various values of "$f_s'$". Quadrant II contains curves which express Eq.(4) for seven different values of "p" and all values of "k" and "p'". Values of "k" are plotted on the vertical axis to the same scale as in Quadrant I, and values of "p'" are plotted on the horizontal axis. Quadrant III. contains graphs of an equation derived from "j" = (1-x/d) in which "x" is expressed in terms of "p" and "p'" by substitution from Eqs. (10), (6), and (4). The values of "j" are plotted on the vertical axis. The curves are for seven values of "p". The values of "p'" are plotted on the horizontal axis to the same scale as in the second quadrant. In Quadrant IV, values of "j" are plotted to the same scale on the vertical side as they are in the third quadrant. Values of the resistance factor, "R", are plotted on the horizontal axis. The curves marked "values of p" are graphs of the Equation "$R_s$" = "$f_s pj$" when "$f_s'$" is a constant equal to 20,000, for seven values of "p". The other lines in this quadrant marked "values of $f_s$" are spaced horizontally to the same scale as "R". These lines are drawn at an inclination which is chosen arbitrarily and they are used to give

"R" for the different values of "$f_s$", as described in the next article on the use of the diagram.

60. <u>Graphical Solution of Formulas: Use of Diagrams.</u> The method of procedure in solving the direct problem will be given first. The external bending moment on the beam is known, the maximum unit compressive stress in the concrete, and the maximum unit compressive and tensile stresses in the steel are specified. The desired quantities are the dimensions of the beam and the amount of steel, compressive and tensile. The latter is given when "p" and "p'" are known, the former may be obtained from the equation, $bd^2 = M \div R$ when "R" is known.

From the curves of Quadrant I, "k" may be determined from any two of the three given stresses. The stresses "$f_s$" and "$f_c$" are chosen to obtain "k" in practically all cases, because of the fact that for all values of "k" shown within the limits of the diagram, "$f_s$" is greater than "$f_s'$" and therefore "$f_s'$" will be safe and need not be considered. Values of "k", greater than shown in the diagram, require an absurdly high percentage of steel, as may be seen from an inspection of Quadrant II. Having determined "k" from "$f_s$" and the "$f_c$" curve, the next step is to refer to Quadrant II. and find values of "p" and "p'" which will give this "k". It will be noted that the "k" ordinate applies to points in several "p" curves. The practice should be to choose

the highest per cent of tensile steel which is practicable. The "p" and "p'" found in this way are then carried to Quadrant III. and "j" is fixed. It remains to find "R". In Quadrant IV. are plotted curves of the equation "R" = 20,000 "pj" for various values of "p". Using the ordinate for "j", and the curve for "p" as just determined, "R" for "$f_s$" = 20,000, is read directly above their intersection. To find "R" for "$f_s$" as given in the problem, the above value must be multiplied by the ration "$f_s$"/20,000. This multiplication may be accomplished graphically by use of the "$f_s$" lines on the diagram, according to the principle of the logarithmic chart. (See Appendix----).

The method is to find the point on the 20,000 line, vertically above in the intersection of "j" and "p", and follow a horizontal line from this point to the specified "$f_s$" line and then read the correct "R" on the vertical.

The above process may seem involved from the length of the explanation, but the following example will illustrate its simplicity.

Example:- Required, the depth and amount of reinforcement of a slab to withstand a bending moment of 2 ,000,000 inch-pounds per 12 inch width, the slab to be made as shallow as possible. The allowable unit stress in the steel is to be 16,000 pounds either tension or compression, and the allowable unit stress in the concrete

is to be 800 pounds.

Using "$f_s$" = 16,000, and "$f_c$" = 800, in Quadrant I., "k" is found to be .377, and "$f_s'$" to be 7100 pounds. This value of "$f_s'$" is safe. Following the horizontal line into the second quadrant, and assuming "p" as 1.4 per cent, "p'" is found to be 1.04 percent. Following the vertical for "p'"=.0104 into the third quadrant until it intersects with the curve for "p" = .014, and reading on the horizontal, "j" is seen to be .882. Traveling on a horizontal with this value of "j" into the fourth quadrant to an intersection with the curve for "p" = .014, thence vertically to the heavy diagonal line marked 20,000, thence horizontally to the line marked 16,000, the value of "R" is read on this vertical to be 202. Now "$bd^2$" ="M" "R", b = 12, and"M"= 2,000,000, and "R" = 202, hence "d" = 28". The area of the tensile steel is .014 x 12 x 28 = 4.7 sq. ins. and the area of the compressive steel is .0104 x 12 x 28 = 3.5 sq. ins.

In the inverse problem, the bending moment, size of beam, and percentages of reinforcement, tensile and compressive, are given and the unit stresses in the materials are required. From the given bending moment and the dimensions of the beam the required resistance factor may be found from the expression "R" = "M" ÷ "$bd^2$". From the given values of "p" and "p'" "k" is determined, using Quadrant II. "j" follows

from quadrant III. The intersection of the line representing this value of "j" and the curve for the given value of "p" in Quadrant IV lies on the vertical corresponding to the value of "R" which would apply if "$f_s$" were 20,000. Call the intersection of this vertical with the diagonal "$f_s$" = 20,000, the point "A". The true value of "R" has been calculated, however, and the value of "$f_s$" is found by locating the intersection of a vertical line at "R" with a horizontal line drawn thru the point "A". Using this stress "$f_s$" and the known value of "k" in Quadrant I, "$f_c$" may be determined.

Example:- A beam is 14 inches wide and 44 inches deep, with 10 square inches of tensile steel and 4 square inches of compressive steel. The depth to the center of gravity of the compressive steel equals 41 inches. The bending moment on the beam is 4,600,000 inch-pounds. Required, the stresses in the concrete and steel.

From the given quantities "p" = $10 \div 14 \times 41^{2}$ = .0174, and "p'" = $4 \div 14 \times 41$ = .007, also "R" = $4,600,000 \div 14 \times 41^2$ = 197. In Quadrant II. the curve for "p" = .0174 (by interpolation) and the vertical "p'" = .007 fix "k" as .426. In Quadrant III., the intersection of the curve for "p" = .0174 (by interpolation) and the vertical, "p'" = .007 fix "j" as .865. The horizontal for "j"=.865 should be followed to its intersection with the "p"= .0174

curve in Quadrant IV. Traveling vertically from this point to the heavy diagonal, "$f_s$" = 20,000 and thence horizontally to the vertical "R" = 197 locates the diagonal corresponding to the desired stress "$f_s$" = 13,300 pounds per sq. inch. Using this stress and "k" = .426, in Quadrant I., gives "$f_c$" as 830 and "$f_c'$" as 7,300 pounds per square inch.

61. Determination of Amount of Compressive Reinforcement for a Required Reduction in Concrete Stress:- Analytical Solution.- As explained in Art. 54, the use of steel in the compression side of a beam is only resorted to, when the depth is limited and the concrete stress exceeds a safe value with all compressive steel omitted. When such a beam is figured for the specified depth and as much tensile steel as it will hold, and the concrete stress is found to exceed the allowable stress, it is often very desirable to have a method for determining the amount of compressive reinforcement necessary to add to bring this stress within the allowable limit. Such a method is given by a formula which may be derived as follows.

The expression for the concrete stress in a beam with tensile reinforcement only, is

$$f_c = \frac{f_s k}{n(1-k)}$$

but $f_s = \frac{M}{A_p j d}$

therefore $f_c = \frac{Mk}{A_p j d\, n(1-k)}$

The symbols in the formulas are the same as usual. For the case where there is compressive reinforcement, the notation is changed to read "$k''$" for "k", "$j''$" for "j", "$f_s''$" for "$f_s$" and "$f_c''$" for "$f_c$". The expression for "$f_c''$" is

$$f_c'' = \frac{f_s'' k''}{n(1-k'')}$$

$$\text{but } f_s'' = \frac{M}{A_s j'' d}$$

$$\text{therefore } f_c'' = \frac{M k''}{A_s j'' d n(1-k'')}$$

The percent of reduction in the concrete stress due to the introduction of the compressive steel is then, calling the per cent "$S_c$"

$$S_c = 100\,\frac{f_c - f_c''}{f_c} = 1 - \frac{k'' j (1-k)}{k j'' (1-k'')} \qquad (A)$$

The tensile steel stress in a single reinforced beam may be expressed as

$$f_s = M \div p j b d^2$$

and in a double reinforced beam as "$f_s''$" = "M" ÷ "pj"bd$^2$",

Therefore, calling "$S_s$" the per cent of reduction of tensile steel stress due to the addition of compressive steel,

$$S_s = \frac{f_s - f_s''}{f_s} = 1 - \frac{j}{j''} \qquad (B)$$

From Eq.(6), Art. 47, it is noted that "k" and consequently "j" is fixed for a given value of "p".

From Eq. (4), Art. 55, it is noted that "k" and consequently "j" depend on values of "p'" for a given value of "p". Hence it is proved that the percent of reduction of the concrete stress, "$S_c$" depends on the percent of compressive reinforcement, "p'", and equation "A" expresses the relation.

Equation "A" will not be found adaptable to ready use because of the number and difficulty of the calculations. For example, if the compressive stress is to be reduced by a percent "$S_c$" and the amount of tensile reinforcement is given by "p", "k" and "j" must be calculated; then "j" must be expressed in terms of "k'" in equation "A" and "k'" found from this equation. From "k'" and "p", "p'" may be determined. All of these steps are laborious. The determination of "S" from given values of "p" and "p'" is not so difficult because Plates I. and III. may be used in finding "k", "j", "k'", and "j'". In solving for "$S_s$" or "p'" in Eq. "B". the calculations are also tedious. The above method of using Eqs. "A" and "B" is outlined to convince the designer of the need for a graphical solution as described in the following article.

62. Graphical Determination:- Equation "A" may be made the basis of a very simple graphical method for determining the amount of compressive reinforcement. Referring to Plate IV., values of "$S_s$" are laid off vertically to the left, and values of "p'" are laid

off horizontally. The upper curve marked "Concrete curves for all values of p" is the average curve for graphs of Equation "A" for different values of "p". In plotting Eq. "A", all the terms in the member on the left were expressed in terms of "p" and "p'". On the right side of the plate, are plotted values of "$S_s$" and the six curves marked "Values of P" are graphs of Eq. "B", (expressed in terms of "p" and "p'"), for these values of "p".

The use of these curves requires very little explanation. When a certain percent of reduction in the concrete stress is required, and the value of "p" is known, the point on the concrete curve horizontally to the right of the required percent is located, and vertically below this point is the value of "p'" necessary. The reduction in the steel stress is then found by reading vertically from "p'" to the given "p" curve, and horizontally to the right.

Example:- A slab with 2% of tensile steel is limited to a 23" depth, the stress in the concrete is found to be 900 pounds per sq. inch for the given loading. Required, the amount of compressive steel to reduce the concrete stress to 700 pounds. The percent of reduction is 22 per cent. This value is located on the left margin of Plate IV. and the horizontal followed to the intersection with the concrete curve; dropping vertically the required per cent of compressive steel

is found to be 0.93. The intersection of the ordinate "p'" = 0.93 with the "p" = .02 curve gives, on the right, 3.7% as the per cent reduction in the tensile steel stress.

63. GRAPHICAL DESIGN OF SLABS:- Construction of Diagram.- In the examples and explanations for the graphical solution of the direct problem in beam design by means of the diagrams on Plates I, II, and III, it was stated that the external bending moment "M", was given and that the resistance factor "R" was the quantity required. As a matter of fact, in all practical problems met with in design, it is necessary to calculate "M" from the fundamental data furnished, and also figure the final dimensions after "R" has been determined. The diagram on Plate V. will enable the designer to perform both of these operations graphically for the case of slabs uniformly loaded. The condition of uniform load on slabs covers the greater number of cases in concrete construction and practically all cases in building work.

The diagram has its basis on the following algebraic relations: Let "l" equal the length of the span in feet, "w" equal the load in pounds per square foot, "$1/_c$" equal a bending moment constant depending on the allowance made for continuous beam action, and "M" equals the bending moment in inch pounds. Also, let "R" equal the "resistance factor" and "d" equal the net depth of the slab. The breadth of "b" of the slab is taken as 12 inches, and the design of this 12 inch width for its

load will of course give the design of the complete slab. If it is desired to use the diagram f r beams more or less than twelve inches in width, it will be necessary to change the loading in the ratio that the actual width has to 12 inches.

It is known that

$$M = \frac{1}{c} W l^2 \times 12 \,\# \qquad (A)$$

when the symbols are expressed in the proper units as given above. Also

$$M = Rbd^2$$

and

$$d = \sqrt{\frac{M}{bR}} = \sqrt{\frac{M}{12R}} \qquad (B)$$

Substituting for "M" from Equation"A"

$$d = l\sqrt{\frac{W}{cR}} \qquad (C)$$

Equation "C" gives the desired depth of the beam in terms of four quantities,"l", "w", "c", and "R". Of these, "l", "w", and "c" are given directly and definitely by the conditions of the problem, and "R" is readily found from the specified unit stresses by the graphical methods given in the foregoing articles. In its logarithmic form, Equation "C" reads log. "d" = log. "l" + 1/2 log. "w" - 1/2 log. "c" - 1/2 log. "R".-- (D)

On Plate V. the five variables are plotted as coordinates and curves in such relations that they may be added and subtracted as shown in Eq. "D", according to the system of the logarithmic chart described in Appendix ----

Values of "R" are laid off on the horizontal axis, and values of "d" are laid off on the vertical axis. Then, for the purpose of plotting the curves for various values of "w", the values for "l" and "$l/c$" were taken as 10 and 1/10 respectively. For example, the curve for "w = 200" is the graph of the logarithmic form of the equation,

$$d = 10\sqrt{\frac{200}{10R}}$$

Now if "c" has a value other than 10, the value of 10R must be increased or diminished accordingly, and this is done graphically by use of the lines for various values of "$l/c$" sloping downward to the right. (It is so arranged that some of the "$l/c$" lines coincide with some of the "w" lines, but this has no bearing on the discussion.) Also, if "l" has some other value than 10, the radical $\sqrt{\frac{200}{10R}}$, must be increased or diminished by means of the lines marked "span in feet" sloping downward to the left. From an inspection of the equation

$$d = l\sqrt{\frac{w}{cR}}$$

and

$$\log d = \log l + \tfrac{1}{2}\log w - \tfrac{1}{2}\log c - \tfrac{1}{2}\log R$$

it is noted that "w", "c" and "R" are to be plotted to the same scale, and "d" and "l" are to be plotted to twice this scale,.

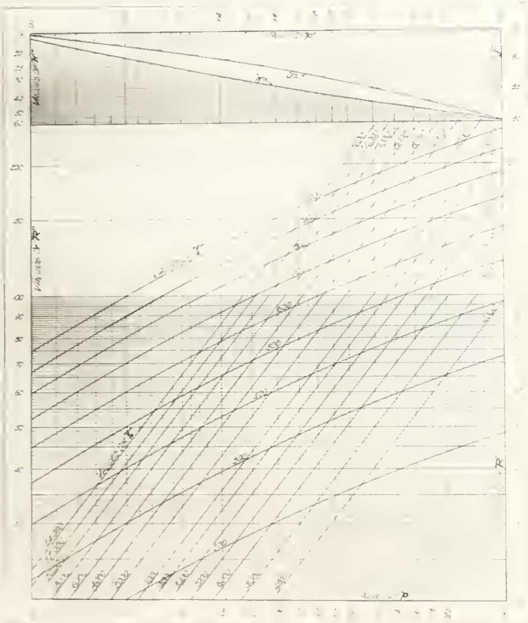
64. Use of Diagram:- In any problem where the usual

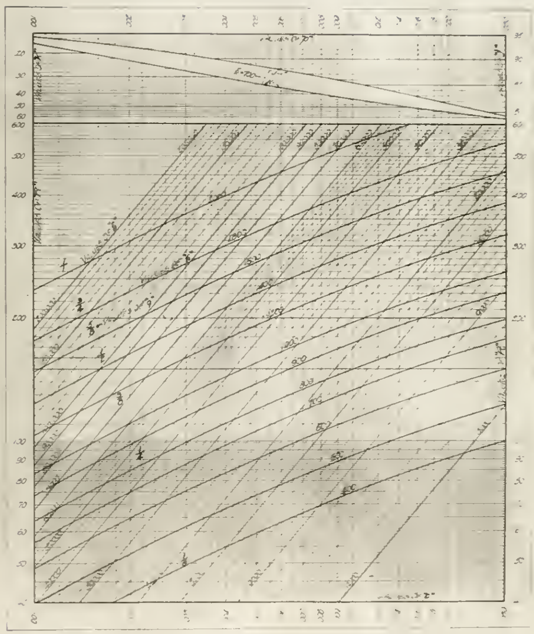
quantities "R", "c", "w", and "l" are given, the procedure is, first, to locate "R" on the lower margin, then to follow the vertical to the first base line "$l/_c$" = 1/10, then to follow the horizontal to the left or right until it meets the diagonal marked with the given value of "$l/_c$", then to move vertically again to the line for "w", after this, horizontally to the heavy base line for "span in feet" equals 10, from this point vertically to the line for the given "l", and finally to the left margin where the required value for "d" may be read.

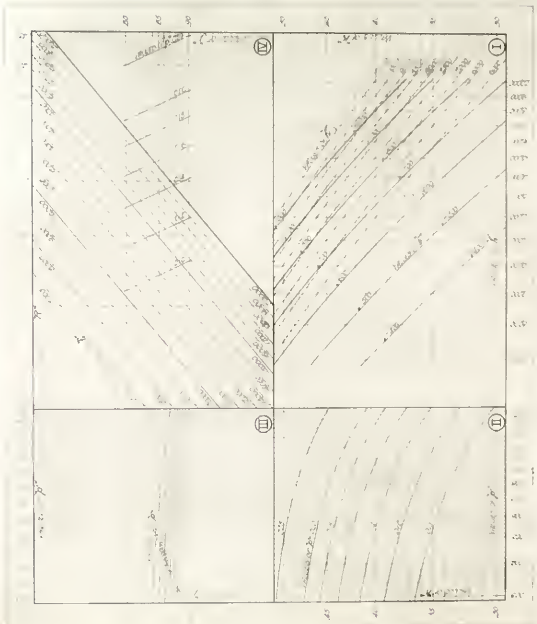
Example:- What is the necessary thickness of a slab of 12 foot span to carry a superimposed load of 150 pounds per square inch? The specifications call for an assumption of 1/12 "$wl^2$"as the bending moment.

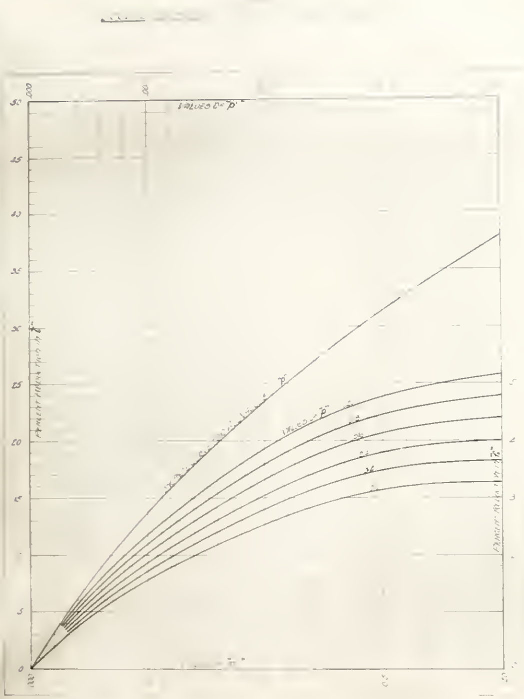
From the unit stresses allowed in this case, it is found that "R" = 120. The problem gives "l" = 12 and "c" = 12. Assuming the slab to be 12 inches thick, "w" = 150 + 150 = 300 Lbs. Traveling along the vertical from "R" = 120, to the first heavy diagonal, then forizontally to the left to the line marked "1/12", then vertically to the line "w" = 300, horizontally to the other heavy diagonal, vertically to "l" = 12, and to the left, to "d" = 5.5. This shows that the assumption regarding the thickness and dead load of the slab was high. Therefore a second assumption is made for

"d" = 6 inches and "w" = 150  75 and the process is repeated and "d" is found to be 4.75 inches. Adding 1.25 ins. for imbedding the steel, makes the depth of the slab 6 inches.






I
II
III
IV


VALUES OF "P"
50
25
20
0

DESIGN OF SLAB.